2021 Undergraduate Research Showcase

Undergraduate Research and Scholarship Showcases

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Comparing Parameters in Growth Models

Payton Lyons
Boise State University

Andrii Zaiats
Boise State University

Trevor Caughlin
Boise State University

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Abstract

Parameters are of significant importance when using non-linearized models. A small shift in one parameter could change an accurate model to an inaccurate one. But to what extent can these parameters change and not affect the accuracy of the model? This question can be answered by setting a specific value in the model to reach and then output the time it took each model to arrive at that value holding all else equal. In these simulations the value used was K/2, which is the half the max population. The initial population and growth rate were the two parameters that were altered over the different simulations of the model. The models had to analytically be solved for time in order to generate the correct output. After running these simulations the relationships the parameters have to the time it takes to reach K/2 is shown. The Gompertz model has steeper contour lines, and seemed to depend less on the initial population and more on the growth rate, whereas the logarithmic model seemed to have a more balanced dependence on both parameters.

Comparing Parameters in Growth Models:

PRESENTER:
Payton Lyons

3D Models demonstrating the effects of a changing growth rate and initial size on population trajectory:

BACKGROUND: Mathematical models of growth are fundamental to understanding population dynamics. Ecological applications of population models are directly relevant to basic and applied sciences, and have key implications for conservation and management policies. However, the non-linear nature of most population models complicates their applications, particularly when inversely deriving unknown quantities from the data. Non-linear functions typically require an understanding and some *a priori* information about the parameters in a form of a bound or statistical prior to constraint the function to a biologically reasonable outcome.

OBJECTIVE

- Aanalyze the difference in how the Gompertz and logistic Growth models behave in response to different combinations of parameters.

METHODS

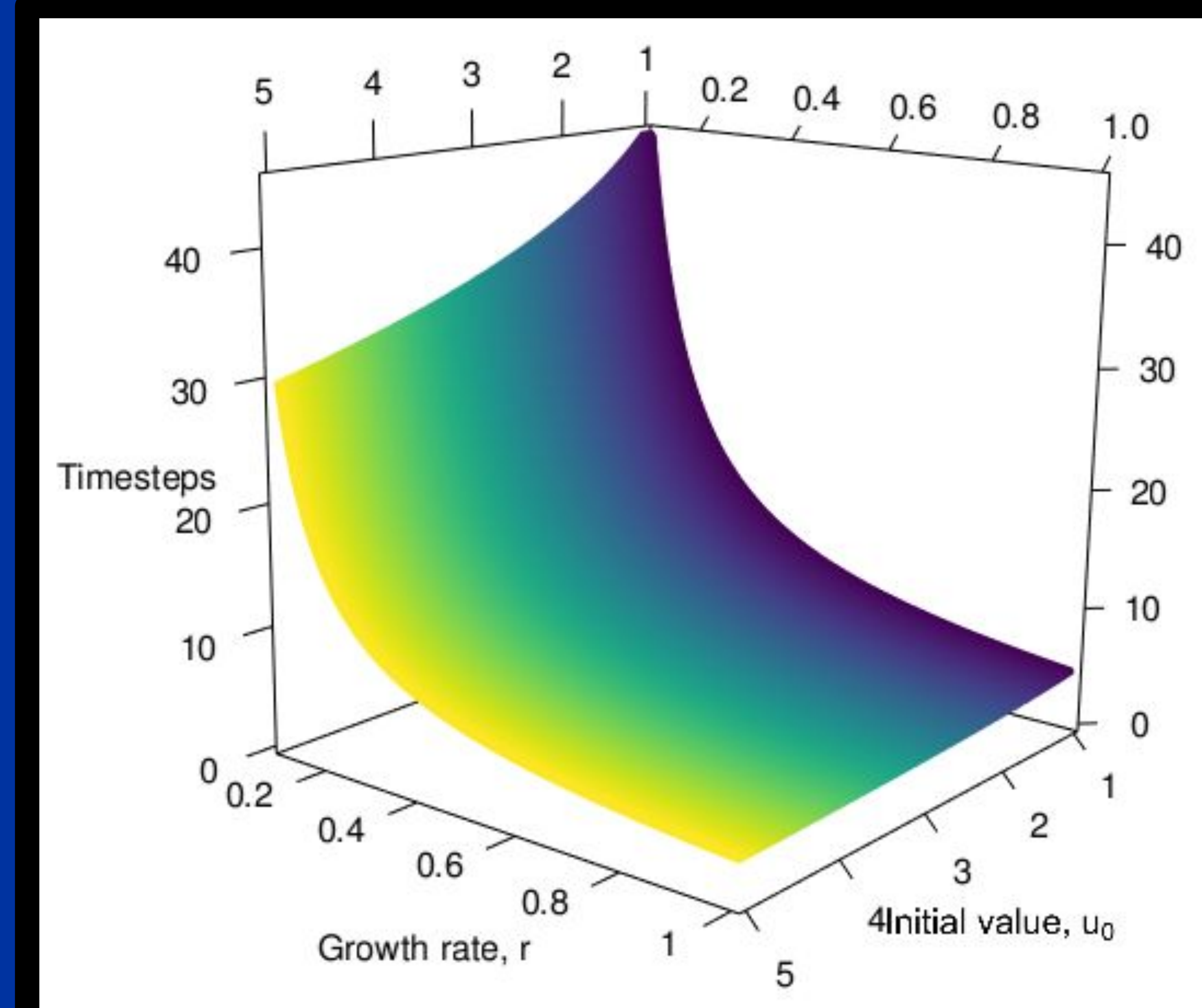
1. Analytically solve the growth models in the biomass and time basis
2. Explore population trajectories over a range of selected parameters.
3. Compare the sensitivity of both the Gompertz and the Logistic growth models in Time and Biomass basis to unknown parameters.

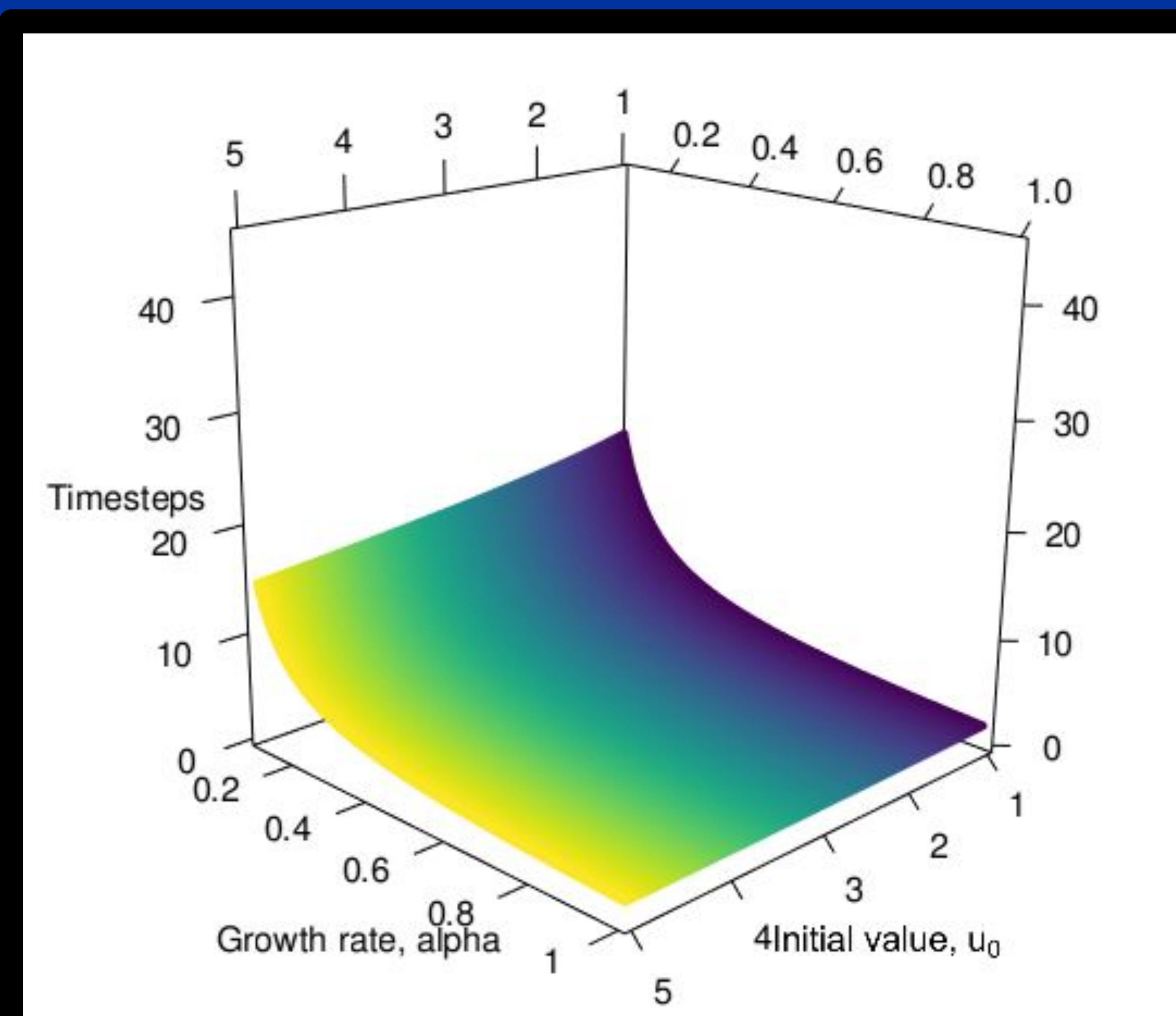
Logistic Growth Model

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

$$t(N) = \frac{ln\left(\frac{N+(K-N_0)}{(K-N)+N_0}\right)}{r}$$

To the right is the 3D representation of the Logistic Growth Model. The vertical axis shows the time a population would take to reach half of it's maximum size as a function of growth rate and initial population size.





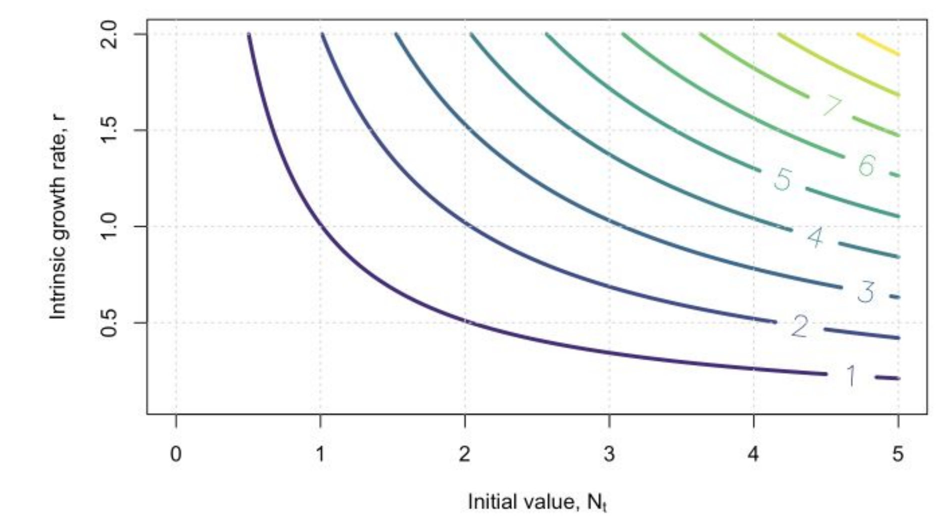
Gompertz Model

$$\frac{dN}{dt} = \alpha N ln\left(\frac{k}{N}\right)$$

$$t(N) = \frac{ln\left(\frac{-ln(\frac{N}{K})}{ln(\frac{K}{N_0})}\right)}{-\alpha}$$

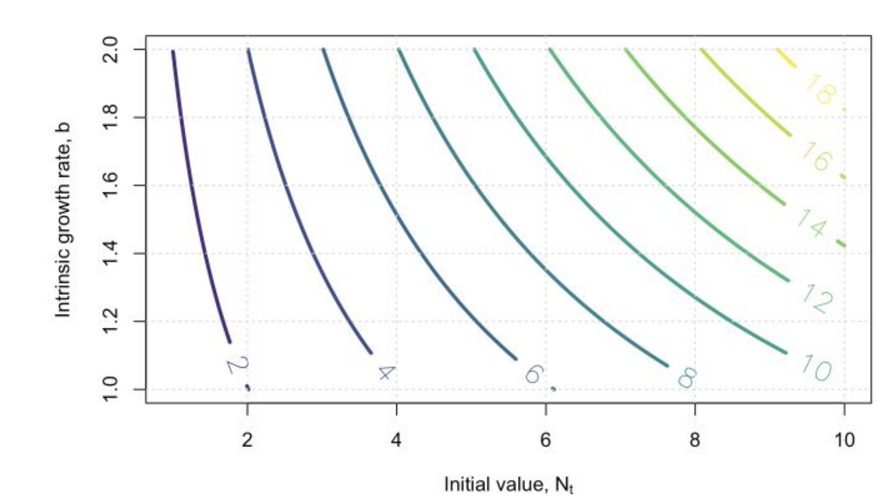
To the left is the 3D representation of the Gompertz model. The vertical axis shows the time a population would reach half of it's maximum size as a function of growth rate and initial population size.

Logistic Growth Model (Discrete Time)



$$N_{t+1} = rN_t(1 - \frac{N_t}{K})$$

Gompertz Model (Discrete Time)



$$N_{t+1} = [b + \alpha log(N_t)]N_t$$

RESULTS

- The Gompertz model shows a smaller dependence on the initial population size, but a larger dependence on the growth rate, which leads to higher initial growth.
- The logistic growth model shows a more proportional dependence on both parameters than the gompertz model.

CONCLUSIONS

In conclusion, both models show a significant dependence on both parameters. However, since the Gompertz model is intended to be asymmetrical it changed less in the latter stages than the Logistic growth model did. This led to the contour lines curving more for the Logistic Growth.

Payton Lyons (paytonlyons@u.boisestate.edu), Andrii Zaiats, Trevor Caughlin
Boise State University

BOISE STATE UNIVERSITY